MATHEMATICAL PRACTICES AS UNDER-DETERMINED LEARNING GOALS: THE CASE OF EXPLAINING DIAGRAMS IN DIFFERENT CLASSROOM MICROCULTURES

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More and more curricula and standards worldwide specify not only mathematical contents as learning goals but also process-oriented goals for mathematical practices. But even with clear formulations in the formal curricula, the implemented curricula of these mathematical practices can diverge substantially for different classroom cultures, as this research report shows for the discursive practice of "explaining". By adopting an interactionist perspective, we compare the implemented curriculum in different video- recorded classroom microcultures. The comparative case study on the topic "explaining diagrams" in grade 5 shows that explaining practices and their underlying norms differ considerably with respect to explanandum, repertory of explanans in epistemic modes, and participation structures.

COMPARING IMPLEMENTED CURRICULA FOR DISCOURSIVE MATHEMATICAL PRACTICES

As process standards on mathematical practices gain an increasing importance in *written curricula* and standards (e.g., CCSS, 2010; KMK, 2004), it is time to ask whether the expectations concerning these learning goals are well defined and whether the *implemented curricula* are comparable in different classrooms. Although some gaps between written and implemented curricula can be found for many learning contents (e.g., van den Akker, 1998), the question is especially important for learning goals that are mainly *orally* established in classroom discourses (not in textbooks), such as explaining, describing, and arguing (that appear as "communicating," for example, in the formal curricula).

Accounting for this *mainly oral status* of mathematical discourse practices, we adopt an interactionist perspective and conceptualize them as being established in the classroom *interaction* (Yackel, 2004). Comparing the implemented curricula for different classroom microcultures therefore means reconstructing the interactively established practices and underlying sociomathematical norms in the interactions.

Our video study focuses on the exemplary discourse practice *explaining*, chosen due to the highest frequency of appearance in the observed classroom discourse in five grade 5 classrooms. In this paper, we use a comparative case study on explaining diagrams to show big differences between two implemented curricula. It raises questions about comparability of learning opportunities for all children, missing preconditions for comparable attained curricula, and difficulties for justice in central exams on a state level.

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THEORETICAL BACKGROUND

Explaining as a classroom practice from an interactionist perspective

In the interactionist perspective, *explaining* is conceptualized as a mathematical practice being interactively established in a classroom microculture and regulated by specific sociomathematical norms (Yackel, 2004). *Learning to explain* in the interactionist approach means successively participating in the explaining practices. The constructs of microculture, norms, and practices allow a shift from evaluating students' utterances as (pseudo-objectively) valid/invalid explanations to those *matching/mismatching* the classroom microculture's norms and practices. This allows capturing of the implemented curriculum in terms of expectations and learning opportunities relative to each classroom. Whereas preceding empirical studies explored the interactionist mechanisms of how practices and norms can be established in principle, our current study intends to specify the explaining practices by systematically taking into account content and epistemic modes.

Distinctions for *explanans* and *explanandum* in the epistemic matrix

We define explaining as a discourse practice that aims at building and connecting knowledge in a systematic, structured way by linking an *explanandum* (the issue that needs to be explained) to an *explanans* (by which the issue is explained). Besides explaining-why, it includes explaining-what and explaining-how. In Prediger and Erath (2014), we developed a conceptual framework for clarifying the addressed mathematical core of the explaining practices in detail. Adopting an epistemological perspective, explaining practices can be distinguished by different logical levels and epistemic modes in the so-called epistemic matrix (see Figure 1). The rows distinguish the *explanandum in 7 logical levels*: the four conceptual levels comprise concepts (categories such as "bar chart"), semiotic representations (here the diagram itself), mathematical models (addressing the relation between reality and mathematical objects/statements), and propositions (mathematical patterns, statements, or theorems); the three procedural levels comprise procedures (such as a general way of drawing a diagram), conventional rules (e.g., "frequencies on vertical axis"), and concrete solutions (such as individual solutions of a task). The columns of the epistemic matrix address the *explanans in six different epistemic modes*: "labelling & naming" is the only mode that can be addressed by a single word (e.g., "maximum").

The mode "explicit formulation" is a linguistically elaborate way to treat an explanandum as it includes definitions and formulating patterns or procedures. The mode "exemplification" addresses examples and counterexamples. The mode "meaning & connection" comprises all aspects of an explanandum that bridge to another level or mode, for example pre-existing knowledge (e.g., meanings, arguments, reasons). The mode "purpose" belongs to a pragmatic approach of explaining by its inner mathematical or everyday functions, for example "by a diagram, we see pattern more clearly." The mode "evaluation" appears in the context of presenting solutions in class.

Explanans in epistemic modes / Explanandum in logical levels	Labelling & naming [l]	Explicit formulation [f]	Exemplification [e]	Meaning & connection [m]	Purpose [p]	Evaluation [ev]
Conceptual levels						
Concepts [C]			#1 #4/6 Markus	#2 Nikolas	#1 #4/6 Markus	
Semiotic representations [R]						
Models [M]						
Propositions [PO]						
Procedural levels		epistemic field of an explanation				
Procedures [P]						
Concrete solutions [S]						
Conventional rules [RU]						

Figure 1: Epistemic matrix for distinguishing explanans and explanandum in explaining practices

In our empirical approach, each explanation that is demanded or given in a classroom interaction can be characterized by its so-called *epistemic field*, that is, the combination of addressed logical level and epistemic mode. Figure 1 contains an exemplary navigation pathway of Episode 1 (see below) in which the teacher addresses the fields – concepts/semiotic representations – "exemplification/purpose" (shortened [CRep]) by asking the class why you can find diagrams more often than lists in printed media. Here, students answer in the expected fields.

DESIGN AND METHODOLOGY OF THE STUDY

The comparison of curricula was led by the following *research questions*:

Q1. Which epistemic fields are addressed in explaining practices?
Q2. How do the explaining practices differ in the navigation between epistemic fields?
Q3.How do students' learning opportunities for explaining practices differ in terms of participation structures?

Data corpus. In the larger project Interpass, video data was gathered in 10 x 12 lessons (of 45–60 minutes each) in five different grade 5 classes. The data corpus also comprised students' and teachers' written products and classroom materials. The small comparative case study presented in this paper focuses on the statistical learning content "diagrams" which was treated in each class in 3–5 lessons. We specifically focus on two classrooms with comparable textbook and student populations; altogether 383 min. of video material, including 111 min. of explaining practices.

Data analysis in four steps. (1) All video data were coded by the applied teaching methods, the epistemic field in which the statistical content "diagrams" was treated, and the emergence of common discursive practices of explaining. (2) All 18 episodes with a common explaining practice in classroom discourse were transcribed and carefully analyzed within their interactive structure. Not only teachers' moves, but also students' answers were classified with respect to the addressed epistemic field and condensed in navigation pathways (see Fig. 1, Fig. 3, and Prediger & Erath, 2014). (3) The navigation pathways in both scenes were contrasted and compared to other scenes for reconstructing typical profiles. (4) For comparing students' learning opportunities, categories were specified for capturing participation structures.

EMPIRICAL CASE STUDY: CONTRASTING TWO CLASSROOMS

Overview of all addressed epistemic fields

Figure 2 shows all the epistemic fields addressed at a certain moment while explaining diagrams. In this first approach to teachers' questions and students' utterances and written tasks, no major differences between the two classrooms can be found.

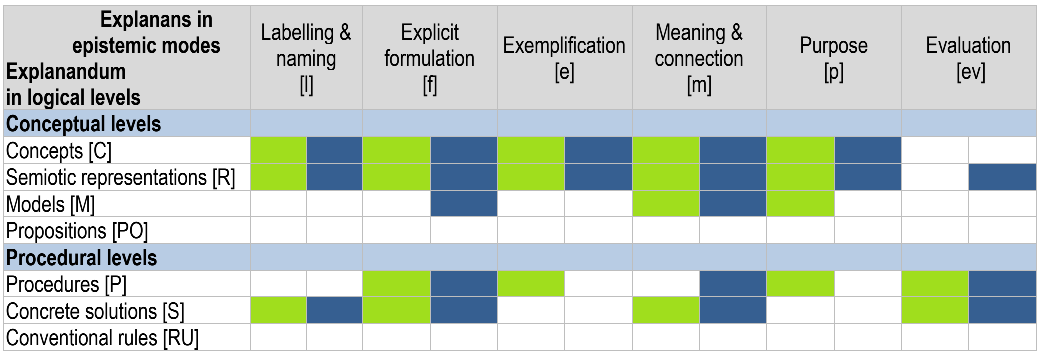

Figure 2: Covered epistemic fields in the classroom of Mr. Maler (brighter green) and Mr. Schroedinger (darker blue)

Deeper analysis of explaining practices

Although on a surface level, both classrooms treat the same learning content "diagrams" and "explaining" in similar epistemic modes, the deeper analysis shows large differences that are illustrated by the two following.

Episode 1: Mr. Schroedinger's classroom: function of diagrams

The teacher (TE) Mr. Schroedinger introduces the topic diagrams with a slide full of examples and constitutes an explanandum on the conceptual level by asking for the function of diagrams [the abbreviated epistemic field is shown next to the transcript].

1	TE	[…] WHY they're doing quite frequently in printed media but also um on TV in the news, um why they're not giving a LIST like that […]	[CRep]
2	Nik	um because maybe because this CATCHES one's eye much faster and um well that you can SEE this faster; so that something is BIGGER; because this is also bigger from its SIZE. So it's MORE because it's BIGGER from its size.	[CRpm]
4	Mar	Because you can CATCH it very fast. For example um now up RIGHT. I think there are such PERCENTAGES; because (that they) CATCH that well it's actually even BETTER than this; (also how many) PEOPLE;	[CRepm]
6	Mar	How many SIBLINGS they have, because then in parts they would maybe have to always go THROUGH our classroom that small	[CRepm]
9	TE	THIS exactly meets the point, these two utterances. THEREFORE you normally do it in the form of such diagrams, because of the clarity actually […]	[CRep]

Nikolas (#2) follows this mode "purpose", but additionally offers a first interpretation of a diagram ("meaning & connection"). The teacher calls further students before summarizing their contributions. Markus first refers to the "purpose" (#4), then to

"meaning" (#4–#6), and finally gives an "exemplification". The teacher asks another student who has nothing to add (non-printed #7/8), then summarizes by recalling those parts of students' utterances that refer to the epistemic modes he initially addressed, namely "purpose / exemplification". The mode additionally addressed by Nikolas and Markus, "meaning & connection", is simply dropped without negative evaluation (whereas in other episodes, these kinds of students' extensions of modes are welcomed by the teacher). The complete navigation pathway is printed in Figure 1. Episode 1 continues with the teacher's initiation of students' individual seat work. Students' written explanations for the difference between diagram and pictogram are later discussed extensively in a whole-class discussion.

The briefly presented Episode 1 could be reconstructed as typical for explaining practices that are often established in this classroom: typical is the location of the explanandum on the conceptual level, the acceptance of different epistemic modes as explanans, and the broad participation of students without immediate single evaluation (cf. Prediger & Erath, 2014, for further examples of the same classroom). In this way, the practice of explaining is constituted as a topic to be learned. Typical for the participation structure in this classroom is also that all contributions are acknowledged and treated as (at least partly) correct.

Episode 2: Mr Maler's classroom: distinguishing names and drawing procedures

Episode 2 starts when the class had collected frequencies of favorite sports and represented them on the blackboard by tally marks and frequency tables.

1	TE	[…] Do you KNOW diff- do you KNOW diagrams? What ARE diagrams, which kinds ARE there, how can this HELP us here; this would be interesting for me now; let's START with- WHO of you actually knows diagrams; MIRKO.	[CRflep]
2	Mir	um BAR CHART does exist.	[CRl]
3	TE	BAR CHARTS, YES bar charts DO exist; what er MAKES UP a bar chart as a bar chart? or differently; how does it LOOK like; Mirko, explain, you said that-	[CRf]
4	Mir	There are no lines of the numbers drawn, but then like well like BARS so to say.	[CRfe]

In #1, the teacher constitutes an explanandum on the conceptual level (concepts/semiotic representations: "diagrams" in general). He initially allows a wide range of epistemic modes: "labelling & naming/explicit formulation/ exemplification/purpose". Mirko (#2) addresses the mode "labelling & naming" by giving only one keyword. In his reaction (#3), the teacher narrows his expectations for epistemic modes and asks Mirko for an "explicit formulation". Mirko fulfills the expected mode and contours his explanation by contrasting bar charts from tally marks ("no lines of the numbers", #4). Mr. Maler's next question shifts the explanandum from the conceptual to the procedural level:

5	TE	Yes, CORRECT; and how, look HOW are they drawn, could you explain to me, how I could DO it maybe for this example	[PSf]

8	Mir	Well, if you would um well, also would write it down like this (only) put away the numbers.	[PSf]
14	Mir	um, well, than you can, well I know (I think there are) TWO bar charts, one up like this and one like er then horizontally	[CRf]

Mirko follows the teacher's navigation after short questions (non-printed, #6/7) and starts his explanation (#8) by referring to the frequency tables on the blackboard. The teacher materializes his description by drawing on the blackboard (non-printed #9–13). Mirko interrupts his explanation of the procedure and navigates back to the conceptual level [CRf] in #14 by mentioning two kinds of charts with horizontal or vertical bars (that have different names in German: bar chart versus "column chart").

15	TE	Very NICE, and now you see we get down to it, MIRKO, ONE of them is called bar chart like you SAID, and the OTHER ONE isn't called bar chart, this we call, does anybody know that? DARIA.	[CRl]
16	Dar	Well, I mean, that, um, this other diagram is often used for watching, for example in politics, for the PARTIES, they go up, or […]	[CRepm]
17	TE	Yes, it is USED quite often at elections; you're completely RIGHT; but first I would like- we maybe just- about to come back to this as well; er to respond to MIRKO again, […] if we're doing it like Mirko just SAID, first write one below the other and then the charts have to- HOW do the charts have to be put there; SO THAT it somehow works; KOSTAS.	[Cl / Pf]
18	Kos	HORIZONTALLY.	[Cl / Pf]
19	TE	HORIZONTALLY, EXACTLY! THEN! you really call it a bar chart; ONLY this is a bar chart.	[CRle]

Mr. Maler continues (in #15) with the explanandum constituted by Mirko, but instead of explaining the meanings, he asks for the correct name of the vertical chart [CRl]. Daria (in #16) does not follow his navigation into the mode "labelling & naming" but mainly refers to "purpose" and "meaning". The teacher evaluates her answer as not matching the intended line of thought and delays it to later (#17). He comes back to his question and navigates it into the field [Pf], the "explicit formulation" for the drawing procedure. Kostas (#18) follows this navigation and names the direction of drawing. This is positively evaluated and the teacher himself answers the question on the names (#19 and later).

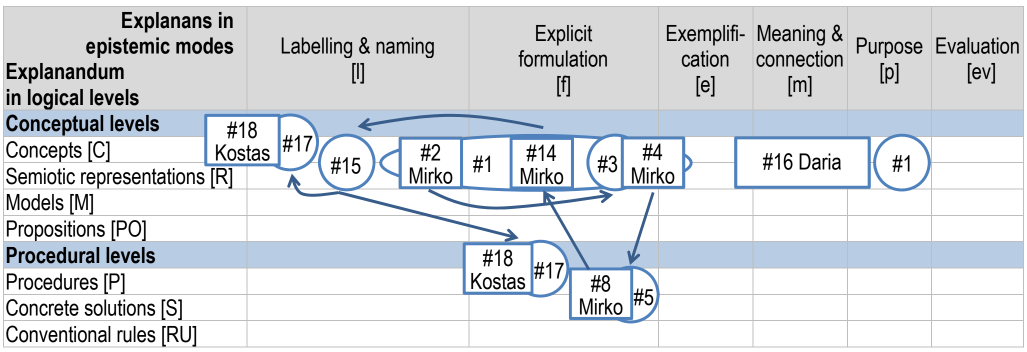

Figure 3: Navigation pathway for Episode 2

Comparing the explaining practices

The briefly presented Episode 2 illustrates typical explaining practices in Mr. Maler's classroom that can be reconstructed as differing in several ways from those found in Mr. Schroedinger's classroom:

- *Explanandum*: Both teachers often treat concrete solutions of tasks, but differ when generalizing: Rather than staying on the conceptual level like Mr. Schroedinger, Mr. Maler shifts the explanandum between conceptual and procedural levels, usually with a strong emphasis on the level of general procedures.
- *Explanans*: Whereas Mr. Schroedinger accepts a large range of epistemic modes, Mr. Maler rejects students' answers in non-expected epistemic modes and constitutes a funnel pattern by successively narrowing his expectations to one or two selected epistemic modes.

Contrasting Mr. Maler's and Mr. Schroedinger's classrooms on explaining diagrams shows that there are distinct profiles for explaining: whereas the explaining practice established in Mr. Maler's classroom can be characterized by the overall profile "explaining procedures with narrow expectations of specific epistemic modes," Mr. Schroedinger's interaction establishes an overall profile of "explaining concepts and models with a wide variety of epistemic modes."

Comparing participation structures, resp. learning opportunities

The comparison of the two episodes shows not only differences in explanans and explandum, but also different learning opportunities in terms of different participation structures: whereas Mr. Maler's typical IRE-sequences often work with one selected student (in Episode 2 with Mirko) and many teacher's explanations, Mr. Schroedinger establishes "IRRRRE-sequences" with many students' replies and reduces his contributions to initiations and summarizing evaluations for all replies.

These participation structures are reflected in the complete unit. In Mr. Maler's classroom, approx. 123 minutes are spent on the topic diagrams, of which about 20% is used for explaining, all of it in oral classroom discourse. But only in about 18% of this explaining time, the students actively explain; the rest is taken by the teacher. In contrast, in Mr. Schroedinger's classroom approx. 260 minutes are spent on diagrams. The 33% of time spent on explanations specifically include about 48 minutes of written explanations in which all students are active. Hence, students are actively involved in 76% of the explaining time.

Although these percentages of a very limited data set can only be interpreted as a very first tendency, these significant differences show that capturing learning opportunities goes beyond the navigation pathways. In our ongoing data analysis, the following categories turned out to be important for analyzing the participation structure: students' active involvement in explaining practices, distinction between oral and

written activities and teaching methods that engage all or only some students, and, finally, division of labor and agency for different epistemic fields.

DISCUSSION AND OUTLOOK

Beyond the close comparability of the classrooms in terms of textbook, student population, and shared formal curriculum, *two divergent profiles* of explaining practices can be reconstructed: “explaining procedures with narrow expectations of specific epistemic modes” versus “explaining concepts and models with a wide variety of epistemic modes.” Hence, it must be doubted whether the students in both classrooms get access to the *same* practice that is mentioned in the written curriculum.

Furthermore, the distinct implemented curricula on explaining are shaped by different participation structures: in Mr. Maler’s classroom, explaining is mainly used by the teacher as a *learning medium* for reaching content goals, whereas in Mr. Schroedinger’s classroom, explaining appears as a *learning content* on which the students get wide opportunities to work, in oral and written form.

Although the ongoing video study will continue to investigate and compare other teaching units for constructing a wider picture, we can already conclude that since already the *implemented* curricula are so different between very comparable class-rooms, we should not be too optimistic for the *achieved* curriculum. As a further consequence, it is an issue of justice to leave the assessed curriculum quite open: How narrow is the norm of explaining that is assessed? And how does it match to the one implemented in the classroom? As a whole, the identified differences in the implementation of the same formal curriculum call for the necessity of widespread professional development for teachers.

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References

KMK. (2004). *Bildungsstandards im Fach Mathematik für den mittleren Schulabschluss: Beschluss der Kultusministerkonferenz vom 3.12.2003*. München: Luchterhand.

National Governors Assoc. Center for Best Practices, Council of Chief State School Officers (CCSS). (2010). *Common core state standards for mathematics*. Washington, DC: CCSS.

Prediger, S., & Erath, K. (2014). *Content or interaction, or both? Synthesizing two German traditions in a video study on learning to explain.* Manuscript in preparation.

van den Akker, J. (1998). The science curriculum: Between ideals and outcomes. In B. Fraser & K. Tobin (Eds.), *Design approaches and tools in education and training* (pp. 1-14). Dordrecht: Kluwer.

Yackel, E. (2004). Theoretical perspectives for analyzing explanation, justification and argumentation in mathematics classrooms. *Journal of the Korea Society of Mathematical Education*, *8*(1), 1-18.